Project

Practical Applications of Analytic (Coordinate) Geometry

Given: Three non-colinear points **A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)**

on a Cartesian Plane

Goal: 1. Constructing triangle ABC with vertices A,B,C, ;

2. Finding :

2.1 Coordinates of the midpoints M_{AB} , M_{BC} , M_{AC} ;

Equations of the :

2.2 Sides AB, BC, AC ;

2.3 Medians AM_{BC} , BM_{AC} , CM_{AB} ;

2.4 Altitudes h_A , h_B , h_C ;

2.5 Right bisectors R_{AB} , R_{BC} , R_{AC} ;

2.6 Finding coordinates of a three special points in triangle:

Centroid, Orthocentre, Circomcentre

Given: A(-4,5), B (6,7), C(2,-5) are vertices of the triangle ABC

2.1 Midpoints $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$: M_{AB}(1,1), M_{BC}(4,1), M_{AC}(-1,0)

2.2 Equations of the sides AB, BC, AC ;

a) The side AB:

1st way

slope: $m_{AB} = \dfrac{7-5}{6-(-4)} = \dfrac{2}{10} = \dfrac{1}{5}$

equation AB in the

y-intersept form: $y = mx + b \Rightarrow y = \dfrac{1}{5}x + b$

for finding b we use point B(6,7)

$7 = \dfrac{1}{5}6 + b \Rightarrow \ b = 7 - \dfrac{6}{5} = \dfrac{29}{5}$

$y = \dfrac{1}{5}x + \dfrac{29}{5} \Rightarrow \ 5y = x + 29$

$$\langle x-5y+29=0 \rangle$$

2nd way

inial equation of a line: $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

equation AB : $\dfrac{y-5}{7-5} = \dfrac{x-(-4)}{6-(-4)} \Rightarrow \dfrac{y-5}{2} = \dfrac{x+4}{10}$

$10(y-5) = 2(x+4) \Rightarrow 10y - 50 = 2x + 8 \Rightarrow$

$2x - 10y + 58 = 0 \Rightarrow x - 5y + 29 = 0$

$$\langle x-5y+29=0 \rangle$$

b) The side BC:

slope: $m_{BC} = \dfrac{-5-7}{2-6} = \dfrac{-12}{-4} = 3$

equation AB in the

y-intersept form: $y = mx + b \Rightarrow y = 3x + b$

for finding b we use point B(6,7)

$7 = 3 \bullet 6 + b \Rightarrow \ b = 7 - 18 = -11 \Rightarrow y = 3x - 11$

$$\langle 3x-y-11=0 \rangle$$

inial equation of a line: $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

equation BC : $\dfrac{y-7}{-5-7} = \dfrac{x-6}{2-6} \Rightarrow \dfrac{y-7}{-12} = \dfrac{x-6}{-4}$

$4(y-7) = 12(x-6) \Rightarrow 4y - 28 = 12x - 72 \Rightarrow$

$12x - 4y - 44 = 0 \Rightarrow 3x - y + 11 = 0$

$$\langle 3x-y+11=0 \rangle$$

c) The side AC:

slope: $m_{AC} = \dfrac{-5-5}{2-(-4)} = \dfrac{-10}{6} = \dfrac{-5}{3}$

equation AB in the y-intersept form: $y = mx + b \Rightarrow$

$y = \dfrac{-5}{3}x + b$. For finding b we use point $C(2,\text{-}5)$

$-5 = \dfrac{-5}{3} \bullet 2 + b \Rightarrow \ b = -5 - \dfrac{-10}{3} = \dfrac{-5}{3} \Rightarrow y = \dfrac{-5}{3}x - \dfrac{5}{3}$

$3y = -5x - 5$

$$\langle 5x+3y+5=0 \rangle$$

inial equation of a line: $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

equation AC : $\dfrac{y-5}{-5-5} = \dfrac{x-(-4)}{2-(-4)} \Rightarrow \dfrac{y-5}{-10} = \dfrac{x+4}{6}$

$6(y-5) = -10(x+4) \Rightarrow 6y - 30 = -10x - 40 \Rightarrow$

$10x + 6y + 10 = 0 \Rightarrow 5x + 3y + 5 = 0$

$$\langle 5x+3y+5=0 \rangle$$

2.3 Equations of the medians AM_{BC}, BM_{AC}, CM_{AB}

a) The median AM_{BC}:

slope: $m = \frac{1-5}{4-(-4)} = \frac{-4}{8} = -\frac{1}{2}$

equation AM_{BC} in the

y-intersept form: $y = mx + b \Rightarrow y = -\frac{1}{2}x + b$

for finding b we use point A(-4,5)

$5 = -\frac{1}{2}(-4) + b \Rightarrow \ b = 5 - 2 = 3$

$y = -\frac{1}{2}x + 3 \Rightarrow \ 2y = -x + 6$

$$\langle x+2y-6=0 \rangle$$

inial equation of a line: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

equation AM_{BC} : $\frac{y-5}{1-5} = \frac{x-(-4)}{4-(-4)} \Rightarrow \frac{y-5}{-4} = \frac{x+4}{8}$

$8(y-5) = -4(x+4) \Rightarrow 8y - 40 = -4x - 16 \Rightarrow$

$4x + 8y - 24 = 0 \Rightarrow x + 2y - 6 = 0$

$$\langle x+2y-6=0 \rangle$$

b) The median BM_{AC}:

slope: $m = \frac{0-7}{-1-6} = 1$

equation BM_{AC} in the

y-intersept form: $y = mx + b \Rightarrow y = x + b$

for finding b we use point B(6,7)

$7 = 6 + b \Rightarrow \ b = 7 - 6 = 1$

$y = x + 1 \Rightarrow \ x - y + 1 = 0$

$$\langle x-y+1=0 \rangle$$

inial equation of a line: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

equation BM_{AC} : $\frac{y-7}{0-7} = \frac{x-6}{-1-6} \Rightarrow \frac{y-7}{-7} = \frac{x-6}{-7}$

$y - 7 = x - 6 \Rightarrow x - y + 1 = 0$

$$\langle x-y+1=0 \rangle$$

c) The median CM_{AB}

slope: $m_{AC} = \frac{6-(-5)}{1-2} = \frac{11}{-1} = -11$

equation AB in the y-intersept form: $y = mx + b \Rightarrow$

$y = -11x + b$. For finding b we use point $C(2,\text{-}5)$

$-5 = -11 \bullet 2 + b \Rightarrow \ b = -5 + 22 = 17 \Rightarrow$

$y = -11x + 17 \Rightarrow 11x + y - 17 = 0$

$$\langle 11x+y-17=0 \rangle$$

inial equation of a line: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

equation CM_{AB}: $\frac{y-(-5)}{6-(-5)} = \frac{x-2}{1-2} \Rightarrow \frac{y+5}{11} = \frac{x-2}{-1}$

$-(y+5) = 11(x-2) \Rightarrow -y - 5 = 11x - 22 \Rightarrow$

$11x + y - 17 = 0$

$$\langle 11x+y-17=0 \rangle$$

2.4 Equations of the altitudes h_A, h_B, h_C

a) The altitude h_A:

slope: $m_{BC} = \frac{-5-7}{2-6} = \frac{-12}{-4} = 3$

since $h_A \perp BC \Rightarrow m_{h_A} = -\frac{1}{m_{BC}}$

equation h_c in the y-intersept form:

$y = -\frac{1}{m_{BC}}x + b \Rightarrow y = -\frac{1}{3}x + b$

for finding b we use point $A(-4,5)$

$5 = -\frac{1}{3}(-4) + b \Rightarrow b = 5 - \frac{4}{3} = \frac{11}{3}$

$y = -\frac{1}{3}x + \frac{11}{3} \Rightarrow x + 3y - 11 = 0$

h_A : $\langle x+3y-11=0 \rangle$

Equation of any line perpendicular to line $ax+by+c=0$
can be represent in the form $bx-ay+p=0$ or $-bx+ay+p=0$
equation of the side BC : $3x-y-11=0$
equation of the altitude h_A that prpendicular to side BC :

$$x+3y+p=0$$

for finding p we use point $A(-4,5)$:
$-4+3\bullet 5+p=0 \Rightarrow p=-11 \Rightarrow$ final equation of the altitude $h_{\grave{A}}$:

$$\langle x+3y-11=0 \rangle$$

b) The altitude h_B:

slope: $m_{AC} = \frac{-5-5}{2-(-4)} = \frac{-10}{6} = \frac{-5}{3}$

since $h_B \perp AC \Rightarrow m_{h_B} = -\frac{1}{m_{AC}}$

equation h_B in the y-intersept form:

$y = -\frac{1}{m_{AC}}x + b \Rightarrow y = \frac{3}{5}x + b$

for finding b we use point $B(6,7)$

$7 = \frac{3}{5}(6) + b \Rightarrow b = 7 - \frac{18}{5} = \frac{17}{5}$

$y = \frac{3}{5}x + \frac{17}{5} \Rightarrow 3x - 5y + 17 = 0$

h_B : $\langle 3x-5y+17=0 \rangle$

Equation of any line perpendicular to line $ax+by+c=0$
can be represent in the form $by-ax+p=0$ or $-bx+ay+p=0$
equation of the side AC : $5x+3y+5=0$
equation of the altitude h_A that prpendicular to side BC :

$$3x-5y+p=0$$

for finding p we use point $B(6,7)$:
$3\bullet 6-5\bullet 7+p=0 \Rightarrow p=17 \Rightarrow$ final equation of the altitude h_B :

$$\langle 3x-5y+17=0 \rangle$$

c) The altitude h_C:

slope: $m_{AB} = \frac{7-5}{6-(-4)} = \frac{2}{10} = \frac{1}{5}$

since $h_c \perp AB \Rightarrow m_{h_c} = -\frac{1}{m_{AB}}$

equation h_c in the y-intersept form:

$y = -\frac{1}{m_{AB}}x + b \Rightarrow y = -5x + b$

for finding b we use point $C(2,-5)$

$-5 = -5(2) + b \Rightarrow b = -5 + 10 = 5$

$y = -5x + 5 \Rightarrow 5x + y - 5 = 0$

$\langle 5x+y-5=0 \rangle$

Equation of any line perpendicular to line $ax+by+c=0$
can be represent in the form $bx-ay+p=0$ or $-bx+ay+p=0$
equation of the side AB : $x-5y+29=0$
equation of the altitude h_C that prpendicular to side AB :

$$5x+y+p=0$$

for finding constant p we use point $C(2,-5)$:
$5\bullet 2-5+p=0 \Rightarrow p=-5 \Rightarrow$ final equation of the altitude h_C :

$$\langle 5x+y-5=0 \rangle$$

2.5 Equations of the right bisectors R_{AB} , R_{BC} , R_{AC}

a) The right bisector R_{BC}:

slope: $m_{BC} = \dfrac{-5-7}{2-6} = \dfrac{-12}{-4} = 3$

since $R_{BC} \perp BC \Rightarrow m_{R_{BC}} = -\dfrac{1}{m_{BC}}$

equation R_{BC} in the y-intersept form:

$y = -\dfrac{1}{m_{BC}}x + b \Rightarrow y = -\dfrac{1}{3}x + b$

for finding b we use midpoint $M_{BC}(4,1)$

$1 = -\dfrac{1}{3}(4) + b \Rightarrow b = 1 + \dfrac{4}{3} = \dfrac{7}{3}$

$y = -\dfrac{1}{3}x + \dfrac{7}{3} \Rightarrow x + 3y - 7 = 0$

R_{BC} : $\langle\ x+3y-7=0\ \rangle$

Equation of any line perpendicular to line $ax + by + c = 0$
can be represent in the form $bx - ay + p = 0$ or $-bx + ay + p = 0$
equation of the side BC : $3x - y - 11 = 0$
equation of the right bisector R_{BC} that prpendicular to side BC :

$$x + 3y + p = 0$$

for finding p we use midpoint $M_{BC}(4,1)$:
$4 + 3\cdot 1 + p = 0 \Rightarrow p = -7 \Rightarrow$ final equation of the right bisector R_{BC} :

$$\langle\ x+3y-7=0 \rangle$$

b) The right bisector R_{AC}:

slope: $m_{AC} = \dfrac{-5-5}{2-(-4)} = \dfrac{-10}{6} = \dfrac{-5}{3}$

since $R_{AC} \perp AC \Rightarrow m_{R_{AC}} = -\dfrac{1}{m_{AC}}$

equation R_{AC} in the y-intersept form:

$y = -\dfrac{1}{m_{AC}}x + b \Rightarrow y = \dfrac{3}{5}x + b$

for finding b we use midpoint $M_{AC}(-1,0)$

$0 = \dfrac{3}{5}(-1) + b \Rightarrow b = 0 + \dfrac{3}{5} = \dfrac{3}{5}$

$y = \dfrac{3}{5}x + \dfrac{3}{5} \Rightarrow 3x - 5y + 3 = 0$

h_B : $\langle\ 3x-5y+3=0\ \rangle$

Equation of any line perpendicular to line $ax + by + c = 0$
can be represent in the form $by - ax + p = 0$ or $-bx + ay + p = 0$
equation of the side AC : $5x + 3y + 5 = 0$
equation of the altitude R_{AC} that prpendicular to side AC :

$$3x - 5y + p = 0$$

for finding p we use midpoint $M_{AC}(-1,0)$:
$3\cdot(-1) - 5\cdot(0) + p = 0 \Rightarrow p = 3 \Rightarrow$ final equation of the right bisector R_{AC} :

$$\langle\ 3x-5y+3=0 \rangle$$

c) The right bisector R_{AB}:

slope: $m_{AB} = \dfrac{7-5}{6-(-4)} = \dfrac{2}{10} = \dfrac{1}{5}$

since $R_{AB} \perp AB \Rightarrow m_{R_{AB}} = -\dfrac{1}{m_{AB}}$

equation R_{AB} in the y-intersept form:

$y = -\dfrac{1}{m_{AB}}x + b \Rightarrow y = -5x + b$

for finding b we use midpoint $M_{AB}(1,6)$

$6 = -5(1) + b \Rightarrow b = 6 + 5 = 11$

$y = -5x + 11 \Rightarrow 5x + y - 11 = 0$

R_{AB} : $\langle\ 5x+y-11=0\ \rangle$

Equation of any line perpendicular to line $ax + by + c = 0$
can be represent in the form $bx - ay + p = 0$ or $-bx + ay + p = 0$
equation of the side AB : $x - 5y + 29 = 0$
equation of the altitude R_{AB} that perpendicular to side AB :

$$5x + y + p = 0$$

for finding constant p we use midpoint $M_{AB}(1,6)$:
$5\cdot 1 + 6 + p = 0 \Rightarrow p = -11 \Rightarrow$ final equation of the right bisector R_{AB} :

$$\langle\ 5x+y-11=0\ \rangle$$

Equations of the medians	Equations of the altitudes	Equations of the right bisectors
AM_{BC} : $x+2y-6=0$	h_A : $x+3y-11=0$	R_{AB} : $5x+y-11=0$
BM_{AC} : $x-y+1=0$	h_B : $3x-5y+17=0$	R_{BC} : $x+3y-7=0$
CM_{AB} : $11x+y-17=0$	h_C : $5x+y-5=0$	R_{AC} : $3x-5y+3=0$

The coordinates of the Centroid P_1:

$$\begin{cases} x-y+1=0 \\ 11x+y-17=0 \end{cases} \Rightarrow \begin{cases} x-y+1=0 \\ + \\ 11x+y-17=0 \end{cases} \Rightarrow \begin{cases} 12x-16=0 \\ 11x+y-17=0 \end{cases} \Rightarrow \begin{cases} x=\frac{16}{12}=\frac{4}{3} \\ 11x+y-17=0 \end{cases} \Rightarrow \begin{cases} x=\frac{4}{3} \\ y=-11x+17=-\frac{44}{3}+17=\frac{7}{3} \end{cases}$$

Centroid: $P_1\left(\frac{4}{3},\frac{7}{3}\right)$

The coordinates of the Orthocentre P_2:

$$\begin{cases} 5x+y-5=0 \\ 3x-5y+17=0 \end{cases} \Rightarrow \begin{cases} 25x+5y-25=0 \\ + \\ 3x-5y+17=0 \end{cases} \Rightarrow \begin{cases} 28x-8=0 \\ 5x+y-5=0 \end{cases} \Rightarrow \begin{cases} x=\frac{2}{7} \\ y=5-5x \end{cases} \Rightarrow \begin{cases} x=\frac{2}{7} \\ y=5-\frac{10}{7}=\frac{25}{7} \end{cases}$$

Orthocentre: $P_2\left(\frac{2}{7},\frac{25}{7}\right)$

The coordinates of the Circomcentre P_3:

$$\begin{cases} 5x+y-11=0 \\ x+3y-7=0 \end{cases} \Rightarrow \begin{cases} -15x-3y+33=0 \\ + \\ x+3y-7=0 \end{cases} \Rightarrow \begin{cases} -14x+26=0 \\ 5x+y-11=0 \end{cases} \Rightarrow \begin{cases} x=\frac{13}{7} \\ y=11-5x \end{cases} \Rightarrow \begin{cases} x=\frac{13}{7} \\ y=11-\frac{65}{7}=\frac{12}{7} \end{cases}$$

Circomcentre: $P_3\left(\frac{13}{7},\frac{12}{7}\right)$