# Distance between skew lines

Given: Two skew lines ($L_1$ , $L_2$) with direction vectors

$\overrightarrow{U}(u_1, u_2, u_3)$ , $\overrightarrow{V}(v_1, v_2, v_3)$ respectively and points

$M_1(x_1, y_1, z_1) \in L_1$ , $M_2(x_2, y_2, z_2) \in L_2$

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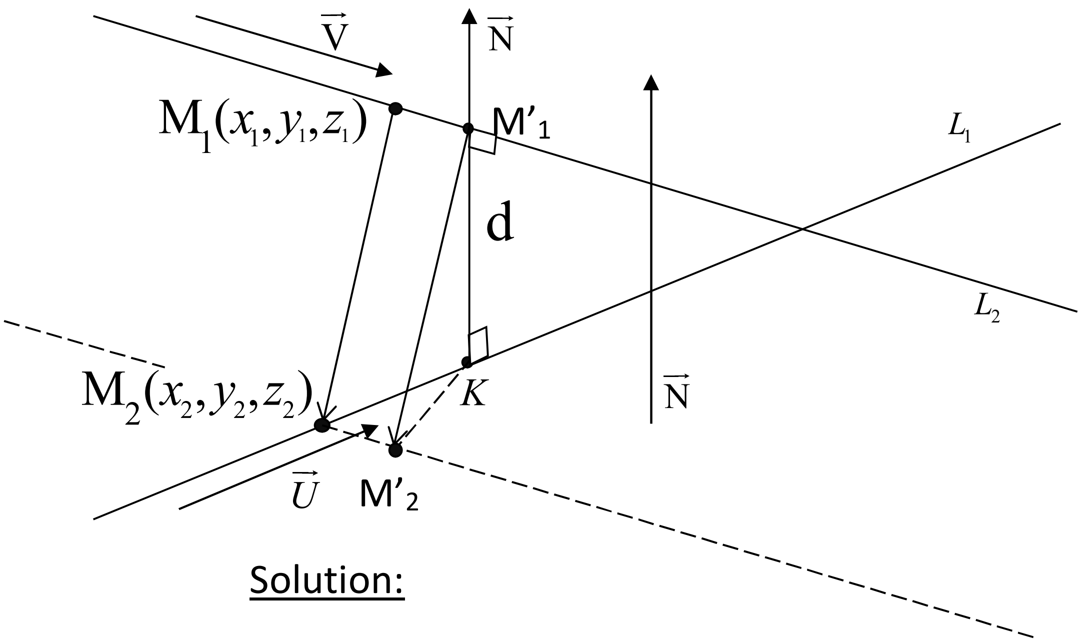
Find the distance $d$ between the lines $L_1$ and $L_2$




<u>Solution:</u>

1) $\overrightarrow{M_1M_2} = \overrightarrow{M'_1M'_2}$

2) $M'_1K \perp L_1$ and $M'_1K \perp L_2$

3) Let: SPR=$\mathrm{Proj}_{\overrightarrow{N}}\overrightarrow{M_1M_2}$ be a scalar projection of a vector $\overrightarrow{M_1M_2}$

onto a normal vector $\overrightarrow{N}$ to the plane Q : $\overrightarrow{N}(A,B,C) = \overrightarrow{U} \times \overrightarrow{V} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$

4) $d = |SPR| = \left|\mathrm{Proj}_{\overrightarrow{N}}\overrightarrow{M_1M_2}\right| = \dfrac{\left|\overrightarrow{M_1M_2} \bullet \overrightarrow{N}\right|}{\left|\overrightarrow{N}\right|} = \dfrac{\left|(x_2 - x_1)\cdot A + (y_2 - y_1)\cdot B + (z_2 - z_1)\cdot C\right|}{\sqrt{A^2 + B^2 + C^2}}$

where $A = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}$, $B = -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}$, C=$\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$

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Final formula

$$d = \frac{\left\| \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ u_1 & u_2 & u_3 \\ V_1 & V_2 & V_3 \end{bmatrix} \right\|}{\sqrt{\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}^2 + \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}^2 + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}^2}}$$