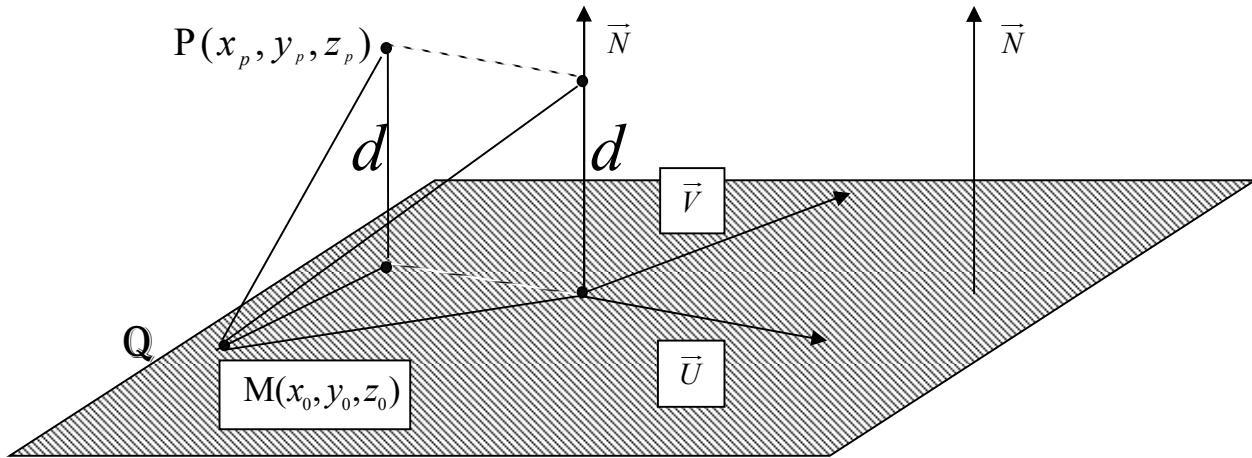


Distance between point and a given plane

Given: Plane Q with two non-collinear vectors $\vec{U}(u_1, u_2, u_3), \vec{V}(v_1, v_2, v_3)$

and point $M(x_0, y_0, z_0)$. Point $P(x_p, y_p, z_p) \notin Q$

Find the distance d between the point P and the plane Q



1) Let $SPR = \text{Proj}_{\vec{N}} \overrightarrow{MP}$ is a scalar projection of a vector \overrightarrow{PM} onto a vector \vec{N}

$$2) d = |SPR| = \left| \text{Proj}_{\vec{N}} \overrightarrow{MP} \right| = \frac{|\overrightarrow{MP} \cdot \vec{N}|}{|\vec{N}|} = \frac{|(x_p - x_0) \cdot A + (y_p - y_0) \cdot B + (z_p - z_0) \cdot C|}{\sqrt{A^2 + B^2 + C^2}} = \\ = \frac{|Ax_p + By_p + Cz_p - Ax_0 - By_0 - Cz_0|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|Ax_p + By_p + Cz_p + D|}{\sqrt{A^2 + B^2 + C^2}},$$

where $D = -Ax_0 - By_0 - Cz_0$

$$3) \text{Normal vector to the plane } Q : \vec{N}(A, B, C) = \vec{U} \times \vec{V} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix},$$

$$\text{where } A = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, \quad B = - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \quad C = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$\text{where } A = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \bullet \vec{i}, \quad B = -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \bullet \vec{j}, \quad C = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \bullet \vec{k}$$

$$4) b = |\vec{U}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

5) Finally we have:

$$h = \frac{A}{b} = \frac{|\vec{PM} \times \vec{N}|}{\sqrt{A^2 + B^2 + C^2}} = \frac{\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_0 - x_p & y_0 - y_p & z_0 - z_p \\ A & B & C \end{bmatrix}}{\sqrt{A^2 + B^2 + C^2}}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \vec{PM}(x_0 - x_p, y_0 - y_p, z_0 - z_p)$$