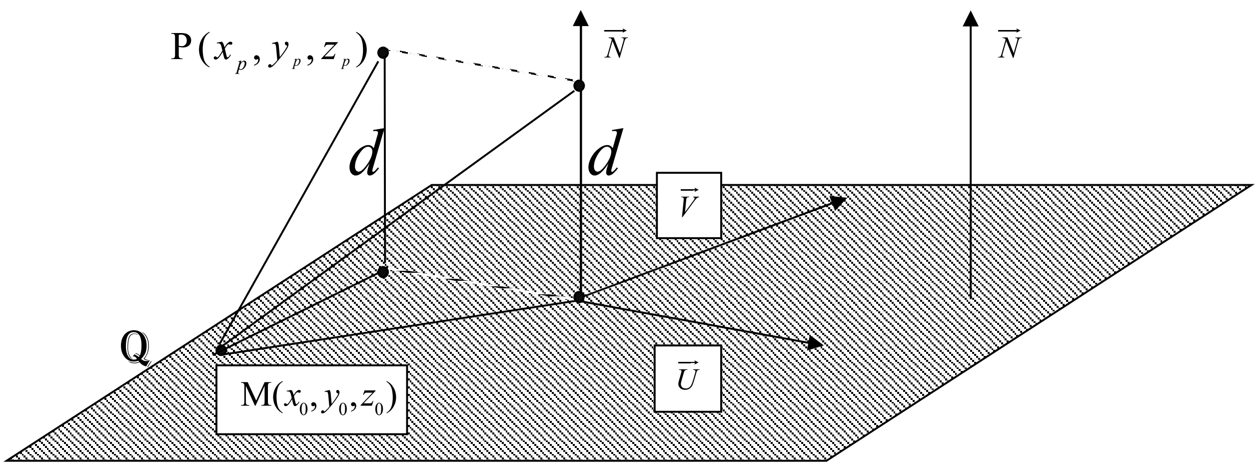
Distance between point and a given plane

Given: Plane $\mathbb{Q}$ with two non-collinear vectors $\vec{U}(u_1, u_2, u_3), \vec{V}(v_1, v_2, v_3)$

and point $M(x_0, y_0, z_0)$. Point $P(x_p, y_p, z_p) \notin \mathbb{Q}$

Find the distance d between the point P and the plane $\mathbb{Q}$



1) Let SPR=$\text{Proj}_{\vec{N}} \overrightarrow{MP}$ is a scalar projection of a vector $\overrightarrow{PM}$ onto a vector $\vec{N}$

2) $$d = |SPR| = \left|\text{Proj}_{\vec{N}} \overrightarrow{MP}\right| = \frac{\left|\overrightarrow{MP} \bullet \vec{N}\right|}{\left|\vec{N}\right|} = \frac{\left|(x_p - x_0)\cdot A + (y_p - y_0)\cdot B + (z_p - z_0)\cdot C\right|}{\sqrt{A^2 + B^2 + C^2}} =$$

$$= \frac{\left|Ax_p + By_p + Cz_p - Ax_0 - By_0 - Cz_0)\right|}{\sqrt{A^2 + B^2 + C^2}} = \frac{\left|Ax_p + By_p + Cz_p + D\right|}{\sqrt{A^2 + B^2 + C^2}},$$

where $D = -Ax_0 - By_0 - Cz_0$

3) Normal vector to the plane Q : $\vec{N}(A, B, C) = \vec{U} \times \vec{V} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$,

where $A = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}$, $B = -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}$, $C = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$

where $A = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \bullet \vec{i}, \quad B = -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \bullet \vec{j}, \quad \mathrm{C} = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \bullet \vec{k}$

4) $b = \left|\vec{\mathrm{U}}\right| = \sqrt{u^2{}_1 + u^2{}_2 + u^2{}_3}$

5) Finaly we have:

$$h = \frac{\mathrm{A}\square}{b} = \frac{\left|\overrightarrow{PM} \times \vec{N}\right|}{\sqrt{A^2 + B^2 + C^2}} = \frac{\left\| \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_0 - x_p & y_0 - y_p & z_0 - z_p \\ A & B & C \end{bmatrix} \right\|}{\sqrt{A^2 + B^2 + C^2}}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \overrightarrow{PM}(x_0 - x_p, y_0 - y_p, z_0 - z_p)$$